NETWORK SYNTHESIS PROCEDURES WITH A POTENTIAL ANALOG COMPUTER

by

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ABSTRACT

This paper is concerned with the operation of a potential analog computer capable of solving many network synthesis problems quickly and easily. The analog device reduces the complex problem of response matching to a relatively simple experimental procedure involving the location of line sources in a conducting medium. These sources are introduced and then manipulated in a well defined fashion until the experimental response, which is displayed on an oscilloscope, matches the desired response. A number of source location techniques are discussed along with the general operating details of the computer. These techniques enable an operator to converge rapidly to a solution regardless of the shape of the curve. The particular method chosen for a given problem would depend upon the shape of the curve and the type of network structure desired. The major portion of the work has to do with the synthesis of networks with prescribed amplitude responses. However, a section is devoted to the operation of the computer for phase response curves.

The experimental results indicate that this computer is capable of producing fast and accurate solutions in minutes, to problems normally requiring days of calculations. In addition, it is possible to examine the phase as well as the amplitude response of a network, thereby greatly extending the usefulness of the device in the field of linear network synthesis.

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I. Introduction

The procedure for synthesizing a linear passive network when the desired response is in the form of an analytic expression is straightforward, although the computation is often extremely involved. However, the required response is often an experimental curve. The procedure for matching a network to such a response curve is considerably more complex, since, in addition to the usual problem of synthesis, it is first necessary to construct a physically realizable function which approximates the curve over the desired range. A number of approximation techniques exist (Ref. 1), but the methods generally require quite lengthy calculations. This paper is concerned with this approximation problem.

A difficult problem can often be solved simply by obtaining the solution to an analogous problem in a different medium. This technique, which is the basis of all modern analog computers, can be applied to network synthesis. Taking advantage of the striking similarity which exists between the expression for the response of a network and the potential due to electric line sources in a conducting medium, the approximation problem can be reduced to a relatively simple experimental procedure. An analog computer, operating on this principle, has been built.

In the past, the emphasis has been on circuit analysis with an analog computer rather than on synthesis. This was due mainly to the fact that no systematic experimental procedure had been found which enabled an operator to converge rapidly to a solution. However, a procedure is presented here which permits rapid convergence to the desired solution regardless of the shape of the response curve. This system, as well as the general operating details of the computer which was used, is described in detail in this report. The theory including the application of logarithmic transformation and general construction details of this computer are discussed in detail in Part I of "The Application of a Potential Analog Computer to Linear Network Synthesis" by Robert Staffin: Thesis for the Degree of M.E.E., Polytechnic Institute of Brooklyn, June 1954 (Ref. 14) and in "Solution of the Approximation Problem of Network Synthesis with an Analog Computer" by S. Lehr, Report R-327-53, PIB-263, June 1953 (Ref. 6) and hence will not be presented in this report.

II. Preliminary Preparation (Amplitude response)

1 - Preparation of the Computer

With the analog computer referred to in the previous section,[(6)(14)] it is possible to obtain the approximate solution to a network design problem. The speed with which this can be accomplished can be attributed, to a great extent, to the simple procedure required to prepare the computer for any given problem. In addition, this procedure is flexible enough to permit the solution of problems requiring different frequency ranges. The following method is recommended:

a) Since the computer sweeps along a logarithmic frequency axis, and since the voltages in the medium are proportional to $\ln|H(p)|$, the curve as it appears on the scope is a log-log plot of $|H(p)|$ vs. frequency. The first step, therefore, is to plot the required response curve on log-log paper.

b) The curve is then traced on to rectangular graph paper.

c) The rectangular plot is now scaled down so that the entire curve can be drawn on the rectangular grid in front of the cathode ray tube.

d) A strip of carbon-impregnated Teledeltos paper is then cut to size, the width corresponding to $\pi/2$ radians or one quadrant.

$$W = \frac{\pi}{2} \quad \frac{L}{\ln 10}$$

where W = width of paper

and L = length of one decade.

e) Conducting strips must be painted on each side in order to improve the electrical contact between the paper and the conducting spring fingers which, in addition to holding the paper in place, also provide a path for return current.

f) The carbon paper is then glued to a piece of graph paper. This not only provides a reference for initial calibration and final readings, but also is effective in keeping the paper flat since the overlapping graph paper can be taped to the cork sheet.

g) The cork sheet, with the paper taped in place, is positioned on the computer so that the rolling probe moves along the very edge.

The machine is now ready for calibration.

2 - Calibration Procedure

The calibration technique consists of two independent adjustments, one for the horizontal scanning, and one for vertical amplification. The first adjustment is merely a matter of setting the horizontal amplifier gain controls and the horizontal centering knob so that the spot on the scope is synchronized with the scanning probe.

For the second adjustment, a half-pole is placed at the origin. This, in the entire p-plane, corresponds to a pole at the origin which represents the impedance of a capacitor. This we know to have a 6db/octave slope. On the log-log plot, this results in a straight line at an angle of 45°. Therefore, the vertical amplifier is adjusted so that with a half-pole at the origin, the curve displayed on the scope is a straight line at a 45° angle.

III. Convergent Location Techniques for Critical Points

Previously, all attempts at network synthesis with an electrolytic tank computer were severely limited. Since no dependable source location technique existed, the manipulation of singularities became an extremely difficult task once the number exceeded four or five. This necessarily restricted the use of the computer to networks with simple response curves. However, a number of techniques will be described here which enable an operator to converge very rapidly to a solution requiring any number of singularities. Removing the upper limit on the number of poles and zeros to be used immeasurably increases the usefulness of the analog computer.

Another tremendous advantage of this computer deserves mentioning at this time. It is possible to obtain certain desired forms by restricting the locations of the singularities. Examples of this would be the use of poles alone to obtain a solution in the form of the reciprocal of a polynomial which is easily synthesized in the form of an LC ladder with a resistive termination (Ref. 8), or the restricting of poles and zeros along the negative real axis, properly paired to obtain an RC ladder. (Ref. 9, 10) The desired form of the solution will help to determine which of the following techniques, or combination of techniques, are to be used for any particular problem.

A given response curve can generally be recognized as a superposition of one, or a combination of the following categories:

a) Curves with an initial or final slope greater than 6db/octave. Fig. MRI-14254-a.

b) Curves which do not vary rapidly with the logarithm of frequency. Fig. MRI-14252-b. This includes straight or nearly straight curves with slopes less than 6db/octave.

c) Curves with a considerable number of maxima or minima in the region of interest. Fig. MRI-14254-c. This category necessarily includes curves which appear to lie within the bounds of classes a) and b) but turn out to be difficult or impossible to match with the methods described for those curves.

The approximation technique peculiar to each of the above classes of curves will be discussed separately. However, there is one fundamental principal common to all, which is responsible for the success of these techniques. Whenever additional singularities are necessary, they should be introduced so that their net effect is negligible upon portions of the curve which have already been matched. The methods for obtaining this effect will be discussed fully in the following sections. But assuming, for the moment, that such methods exist, they then suggest a simple procedure for matching a given curve.

Starting on the extreme left, or low frequency side, one or two singularities are introduced and then manipulated so as to match the curve over a small range of frequencies. Where they are placed will depend upon which of the aforementioned categories this portion of the curve coincides with. Then, progressing from left to right, singularities are added at each critical frequency in turn, so that the curve is affected about that frequency and possibly to the right, but not to the left. New poles and zeros are introduced only after the curve has been perfectly matched over the frequency range in the neighborhood of, and to the left of the last group of singularities added. With these few ideas in mind, we can now proceed to the more detailed analysis of the three previous classes of curves.

1) <u>Curves with an initial or final slope greater than 6db/octave.- Fig. MRI-14254-c</u>

This type of curve can often be matched with poles alone, resulting in the easily synthesised form

$$H(p) = A \left[\frac{1}{p^m + b_{m-1}\, p^{m-1} + \ldots\ldots + b_o} \right] \qquad (1)$$

the technique used for this type of synthesis will be most easily understood if we consider the response due to one pole in different positions. For illustrative purposes, the conducting paper with the singularities will be drawn approximately as they would appear on the computer, below each response curve. Fig. MRI-14255.

The half pole in position 1 (representing a full pole on the negative real axis) results in the standard, single stage, RC filter characteristic which approaches 6db/octave at high frequencies. The response to a full pole eventually drops off at 12db/octave regardless of where it is placed in the plane. However, the value of θ, or in other words, the distance from the real frequency axis determines the shape at the knee of the curve. If a full pole were placed at position 1 (corresponding to a double pole on the negative real axis) the result would be a broad curve similar to the RC characteristic shown but with a final slope of 12db/octave. As the pole moves away from position 1, the curve tends to break more sharply until, in the neighborhood of position 2, the response is almost flat to the left of the singularity, breaking very rapidly thereafter into 12db/oct. From position 2 to position 3, the curve ceases to be monotonic. The peak which appears in the neighborhood of the singularity, increases in height and, in the limit as the pole moves onto the real frequency axis, goes to infinity. (These curves are accurately drawn in E. Gorczycki's thesis; Ref. 7).

When viewed in the light of the previous discussion, the response curves for a pole assume special significance. The response is level in the low frequency region. This is apparent from equations (2) and (3), (see Fig. MRI-14255) if we let $\omega << \beta$, that is,

$$|H(j\omega)| \cong \frac{1}{\alpha^2 + \beta^2} = \text{constant; for } \omega << \beta.$$

Therefore, the introduction of a single pole merely adds a constant potential to the left side of the curve. Thus, the portion to the left of the singularity remains relatively undisturbed. The effect is most striking when a pole is placed in position 2. For this case, the curve is flat and monotonic over the greatest range of frequencies. Position 2 can be determined in the following manner. Rewrite equation (3).

$$|H(j\omega)|^2 = \frac{1}{(\alpha^2 + \beta^2)^2} \left[\frac{1}{1 + 2\dfrac{(\alpha^2 - \beta^2)}{(\alpha^2 + \beta^2)^2}\,\omega^2 + \dfrac{\omega^4}{(\alpha^2 + \beta^2)^2}} \right] \qquad (4)$$

For the condition of maximal flatness, as many derivatives as possible are made to vanish at the origin. This is equivalent to equating successive coefficients of the ω^2 term in the numerator and denominator of (4). For this case,

$$\frac{2(\alpha^2 - \beta^2)}{(\alpha^2 + \beta^2)^2} = 0$$

$$\alpha^2 - \beta^2 = 0$$

$$\alpha = \beta; \quad \alpha = -\beta$$

Thus, position 2 occurs at $\theta = \frac{3\pi}{4} = 135^\circ$ the introduction of a pole at this point must therefore have the least effect on an established response. This effect was utilized in the examples which will be presented illustrating this type of technique.

When the required response curve appears to lie within class (a), the following procedure is recommended: First, the final slope should be estimated. This will indicate the total number of full poles eventually placed in the conducting paper since the final slope will approach 12n db/oct. where n is the number of full poles in the paper. The singularities are then added from left to right until the desired response is obtained. However, as indicated previously, the poles should be added only after the previous singularities have been adjusted for the best approximation in the lower frequency regions. After the proper number of singularities have been added, the curve should be very close to the desired shape. If necessary, small corrections can be made before the final pole positions are noted.

Two illustrative examples will now be presented. Between them, they not only illustrate the several points discussed in connection with the above technique, but they also indicate in a striking manner, the accuracy of the computer and the speed at which a solution may be arrived at.

Example No. 1

A network was required which was to have a linear response on a rectangular plot of Gain vs. Frequency. Fig. MRI-14256. It was therefore necessary to find an analytic expression capable of approximating this curve. The curve was first replotted on log-log coordinates. The resulting curve, Fig. MRI-14257, which was plotted over four decades, appeared to fall within class (a). An attempt was made, therefore, to synthesize it as the reciprocal of a polynomial.

The poles were placed exactly in the manner described previously. The multiple photographs in Fig. MRI-14258 show the curve in successive stages as each singularity was added from left to right. The response to the singularities on the negative real axis alone is shown in Fig. MRI-14257 (broken line). This response, when referred again to rectangular coordinates is represented by curve (a) Fig. MRI-14259. It will be noted that the approximation to the left of f = 0.7 is excellent, but to the right, the curve does not fall sharply enough. This, then, is the perfect situation for a pole in the neighborhood of position 2 since, as was explained, this will not affect the curve to the left and will add 12db/octave slope on the right. The addition of this pole (encircled on Fig. MRI-14257) resulted in curve (b) Fig. MRI-14259. Increasing the order of this pole (curves (c) and (d)) improves the approximation but increases the complexity of the final network. Since the final slope for this curve is infinite, the number of poles necessary would depend upon the allowable tolerance and not upon the criteria expressed in the previous section. The network function and the resulting network are shown in Fig. MRI-14259. The total time required to locate the critical points for this problem was less than thirty minutes.

Example No. 2

The network for this example was to meet the following requirement. The logarithm of the transfer impedance Z_T was to be linear with respect to angular frequency, Fig. MRI-14260. After replotting on log-log coordinates Fig. MRI-14261, the curve appeared to have a final slope of approximately 36db/octave. The number of full poles in the final solution should then be in the neighborhood of three. A half pole deviation in either direction is not too critical.

This problem was solved with the same technique used for example 1. The poles on the negative real axis were used to shape the knee of the curve and the pole at position 2 Fig. MRI-14261 gave the required final slope without affecting the shape already established. Again, the multiple photographs, Fig. MRI-14262, picture the successive introduction of singularities from left to right.

The final result, replotted on semi-logarithmic coordinates, can be seen in Fig. MRI-14263. This result, which was obtained even more rapidly than the previous solution, is certainly indicative of the capabilities of the analog computer described in this paper.

2) Curves which do not vary sharply with the logarithm of frequency

Curves within this class can often be matched with singularities restricted to the negative real axis. This restriction greatly simplifies the matching technique, but even more important is the fact that by pairing the poles and zeros properly, it is possible to obtain a solution which will always synthesize to an RC structure. Therefore, for the discussion of this technique, it will be best to examine the properties of a pair of singularities, a pole and a zero, placed side by side on the negative real axis, Fig. MRI-14264-a.

The response to a zero on the negative real axis (curve 1) is identical to that of a pole (curve 2) except that it is inverted. The superposition of these responses results in a curve level at both ends, and with a positive or negative slope depending upon whether the zero or the pole is closer to the origin. Adding another pole and zero to the right would result in a curve with a double step. Fig. MRI-14264-b illustrates the stepped effect due to a number of poles and zeros placed along the axis. As the singularities are brought closer together, the response tends to develop into a smooth curve.

In order to match a curve with this technique, it is again advisable to start from the left. As illustrated in Fig. MRI-14264-b, the response is broken and then leveled off creating a stepped effect which approximates the required curve. If a close approximation is required, the steps should be made smaller by placing the singularities closer together. The following example is an excellent illustration of this method.

Example No. 3

An RC network was required which was to have the same amplitude response as the input impedance of a "spiral-four" transmission line. The frequency range was to extend from 10 c/s to 100,000 c/s. The curve, obtained from experimental data and plotted in Fig. MRI-14265, clearly belongs to class (b). Since an RC driving-point impedance was desired, the poles and zeros along the negative real axis were required to alternate in order to assure physical realizability.

The multiple photographs Fig. MRI-14266 show the curves being broken and leveled off as each singularity is added. It should be noted that the introduction of successive singularities does little to disturb the low frequency portion of the curve. The results for this example are presented in Fig. MRI-14267.

3) Curves with a considerable number of maxima and minima in the region of interest.

In order to match curves within this category, it becomes necessary to introduce a third method for placing singularities in the conducting medium. A pole-zero configuration would be ideal if it had a great localized effect while still satisfying the requirement that it leave undisturbed, portions of the curve already matched. This suggests the use of a pair of singularities, a pole and a zero, closely spaced, so that the net effect in all but the immediate vicinity of the pair is neglibible.

In Fig. MRI-14268, the response is drawn for such a pair of singularities, separated by a distance δ, and at a distance d from the scanning axis. For ω removed from the vicinity of the singularities, $r_o \cong r_x$ and the resultant field from the two opposing sources is approximately zero. Therefore, the curve is flat for all but a small band of frequencies in the neighborhood of the pair where the above approximation is not valid. If δ remains fixed but d is allowed to vary, the height h of the curve varies from a negligibly small value when d is a maximum, to infinity when d goes to zero. However, Δf, the range over which the singularities have an effect, remains relatively unchanged. If d is kept fixed but δ varies, then the effect on the height of the curve is small compared to the variation in Δf. For δ very small, $r_o \cong r_x$ is valid for all frequencies except those directly in line with the singularities. For large values of δ, the approximation is invalid over a larger band of frequencies. It is apparent from Fig. MRI-14268, that for a maxima, the pole must be placed closer to the scanning axis, while for a minima, the reverse is true.

The procedure for matching curves in class (c) often involves the use of the techniques developed for classes (a) and (b) as well. Large portions of the curve can often be matched with the simpler methods described in the previous sections, in fact, a slowly varying curve with a few small peaks and valleys can be approximated by a smooth curve developed in the manner discussed for class (b) with the peaks and valleys superposed upon this curve by the addition of properly placed pole-zero pairs. However, for curves with a large number of closely spaced maxima and minima, the standard procedure is recommended. Proceding from left to right, after preliminary shaping with methods (a) or (b) if necessary, a pole-zero pair is introduced in line with the first maxima or minima. Assuming a maxima appears first, the pole is placed closer to the axis and the two singularities are then manipulated until the curve is matched in the neighborhood of, and to the left of this maxima. The next pair is then introduced in line with the minima, this time with the zero in the dominant position. These are manipulated until the minima is completely formed. New singularities are placed in the medium only after the previous ones have been fully adjusted. This process continues until all the peaks and valleys have been accounted for.

The photographs in Figs. MRI-14269 and MRI-14270, illustrate the effectiveness of the pole-zero pairs. Fig. MRI-14269 is a series of pictures showing the superposition of peaks and valleys on a smooth curve. Fig. MRI-14270 illustrates the standard method progressing from left to right.

One additional point should be made before leaving the subject of source location techniques. After a solution has been obtained, it is sometimes possible to remove certain singularities and to readjust the remaining ones to preserve the match. This economization should certainly be attempted after each solution since it may result in a substantial reduction in the complexity of the final network.

IV. Phase Response

The application of a potential analog computer to problems involving phase response is still in a primitive stage. However, the machine used for this report is capable of giving a satisfactory phase display.

In order to display phase, it is necessary to measure a differential voltage normal to the $j\omega$ axis and to integrate the result (Ref. 14). This would normally involve the use of a double rolling probe. Such a probe may present mechanical difficulties and was not attempted in this investigation. Instead, advantage was taken of the symmetry conditions discussed in Ref. 14 (Part I Section II (5)), so that it was possible to display phase with the same probe used for the amplitude response. In that reference, it was shown that by replacing half the conducting medium by an equipotential grounded strip, the phase was merely multiplied by a scale factor, a constant which is automatically set during calibration. The difference in potential between a single probe and this grounded strip is then equivalent to the potential across the double probe. The potential difference is fed to a d-c integrating amplifier. The output of the integrator goes to the vertical d-c amplifier on the oscilloscope. The sweep is synchronized with the carriage movements the same as for the amplitude response. The display on the scope is therefore a plot of phase angle as a function of the logarithm of frequency.

In order to obtain satisfactory integration, the time constant of the integrating circuit should be at least ten times the period of the function to be integrated. Since the carriage requires five seconds to completely sweep from one end of the paper to the other, an integrator with a minimum time constant of one minute must be used. A d-c integrating amplifier can meet the long time constant requirement without a loss in signal strength, in fact, with a proper choice of components, the circuit can introduce additional gain.

The intergrating amplifier for this computer (Ref. 13) has an effective time constant of approximately six minutes and an overall gain of two. The circuit diagram is presented in Fig. MRI-14271. The operation of the integrating amplifier and the rest of the computer in order to obtain a phase display, is discussed in the Appendix.

1 - Preparation of the Computer

The preparation of the computer for phase response is very similar to the steps taken for amplitude response.

a) The response curves should be plotted on semi-logarithmic coordinates.

b) The curve is then traced onto rectangular graph paper.

c) The rectangular plot is then scaled down and drawn on the grid in front of the cathode ray tube.

d) A strip of Teledeltos paper is then cut to size. However, it should be one-half inch wider than for the corresponding amplitude response.

e) Conducting stripe are painted on each end, and in addition, a half inch conducting strip is painted along the entire length of the paper.

f) The carbon paper is then glued to the graph paper and taped down to the cork sheet.

g) The paper is positioned on the computer so that the pickup probe moves along a line approximately three-eights of an inch away from the grounded strip.

After this procedure, the computer is ready for calibration.

2 - Calibration Procedure

The calibration technique again involves two independent adjustments. The horizontal sweep is set so that the scanning spot and the carriage move in unison (see Part II Section I (2)). For the vertical adjustment, a known phase response is set up on the computer and the vertical gain and centering controls are adjusted. However, unlike the amplitude display, once the centering control is set, it must remain fixed throughout the operation.

The curve used for calibration is the phase response for the input impedance of a parallel RLC circuit. This network is characterized by a pair of conjugate complex poles; in the analog, this corresponds to a pole placed at some point in the conducting medium. The phase of this network

is asymptotic to $\pm$ 90°, approaching + 90° as ω goes to zero, and - 90° as ω goes to infinity. Therefore, in order to calibrate for phase, the vertical amplifier is adjusted with a pole in the middle of the conducting medium so that the upper and lower asymptotes are separated by a distance equal to 180°. This distance is arbitrary and may be selected to conform to any convenient scale. The series of photographs in Fig. MRI-14272 represent a set of calibration curves. The different curves correspond to a single pole at various distances from the real frequency axis.

3 - Experimental Results

No attempts were made to synthesize networks for prescribed phase characteristics. However, a series of phase curves were displayed and photographed. These photographs are presented in Figs. MRI-14272 and MRI-14273. Fig. MRI-14272 contains the aforementioned set of calibration curves, and Fig. MRI-14273 represents, in order, the phase response for each of the examples in the preceding section.

These pictures serve to show that the computer is capable of producing a clear, steady phase response curve, thereby opening the door not only to synthesis for phase, but possibly the simultaneous display of both phase and amplitude.

V. Conclusion

The potential analog computer is capable of solving many complex network synthesis problems. When used in conjunction with the source location techniques discussed in this paper, it is possible to converge to a solution in considerably less time than the usual computational methods employed for response matching. In addition, the experimental results indicate that in spite of the speed of operation, a relatively high degree of accuracy can be obtained.

The ability to display phase greatly increases the usefulness of the computer, and introduces the possibility of the simultaneous display of both amplitude and phase. This would certainly be a major step forward in the field of linear network synthesis.

APPENDIX

D. C. Integrator Circuit for Phase Response

Figure MRI-14274 is a diagramatic sketch of the analog computer. For amplitude measurements, the rolling probe is connected directly to the oscilloscope. For phase displays, an integrating amplifier is placed between the probe and the scope as indicated in the figure. A detailed description of the computer can be found in Ref. (6) (14). In this section, we will be primarily concerned with the operation of the integrating amplifier.

The amplifier itself is a d-c amplifier with a gain of about 700, Fig. MRI-14271. The series resistor and the shunt capacitor transform the amplifier into a long time constant integrating circuit. A microswitch is connected directly across the terminals of the capacitor. This switch, which closes simultaneously with the left reversing switch, completely discharges the capacitor before each run. Therefore, the operation of the computer for phase measurements is almost identical to that required for the amplitude display except for one additional step, the adjustment of the integrating amplifier.

The following procedure is the one that was found to be most effective:

1 - The amplifier requires two power supplies, a plus 250 volt supply and a minus 300 volt supply. Since the grid bias for some stages are obtained from a voltage divider connected between these two supplies, it is necessary to turn the negative supply on first in order to prevent the grid voltages from becoming excessively positive and ruining the tubes. The first step, therefore, is to turn on the power supplies, the negative one first, and to set the voltages to the required values. A voltmeter should be used since the meters on the power supplies are often inaccurate.

2 - The voltage on the grid of the third stage is extremely critical. In order to aid in the adjustments, a terminal which is connected to this grid, was placed on the front of the integrator. Thus, after adjusting the power supply voltages, the voltmeter should be clipped to this terminal.

3 - The final voltage on this grid should, when properly adjusted, be in the neighborhood of - 3 volts. Therefore, the following rough adjustments should be made. With the voltmeter on the grid terminal, potentiometer No. 1 should be adjusted to bring the grid voltage into the - 3 volt range. However, occasionally, the grid will appear to be "captured" due to the effect of grid current. When this occurs, the potentiometer will have no effect. In this situation, the voltage on the negative power supply

should be slowly increased. The grid will then suddenly go very negative. The voltage can then be brought back to the - 3 volt region with potentiometer No. 1 or by again reducing the negative supply voltage.

4 - At this point, the circuit is capable of integrating an applied signal. With no probes in the paper, the response should be horizontal line. However, one or two runs will indicate that the integrator is charging the condenser in a manner identical to that which would occur if a constant potential had been applied to the input of the amplifier. This results in a sloping, rather than a horizontal line, thereby introducing a serious error in the display. This is due to the fact that the aforementioned grid voltage has not been finely adjusted. In order to correct for this, it is necessary to resort to potentiometer No. 2.

5 - It is easier to adjust potentiometer No. 2 with the carriage stationary. A poor adjustment will then tend to move the spot in a vertical line, either up or down, depending upon the direction in which the capacitor is charging. If the spot moves off the screen, it can be brought back by depressing the shorting switch. The procedure then consists of shorting the capacitor and then adjusting potentiometer No. 2 until the spot remains fixed. This is essentially a fine adjustment on the grid voltage. If the potentiometer is not capable of fixing the spot, a slight change in potentiometer No. 1 will bring the voltage into the range of potentiometer No. 2. This adjustment should be checked periodically since the power supplies tend to drift slightly. Generally, a slight adjustment of potentiometer No. 2 will be all that is necessary. The computer will then be set for a phase display.

One or two additional things should be noted. First, the display is valid only for the period during which the carriage moves from left to right. This is due to the fact that the capacitor continues to charge on the reverse trip. However, this can possibly be overcome by reversing the polarity of the poles and zeros. The capacitor would then tend to discharge at a rate identical to the charging rate. Thus, the return trace would be identical to the original response.

The second point is concerned with the handling of poles and zeros on the $j\omega$ axis. A probe placed right on the axis would have no effect since it would be grounded by the equipotential strip. Thus, a correction factor of $\pm 180^\circ$ would have to be added for each pole or zero on the axis. However, the need for a correction can be eliminated by placing the singularity very close to, but not quite on the $j\omega$ axis. This will result in a very close approximation to the correct response.

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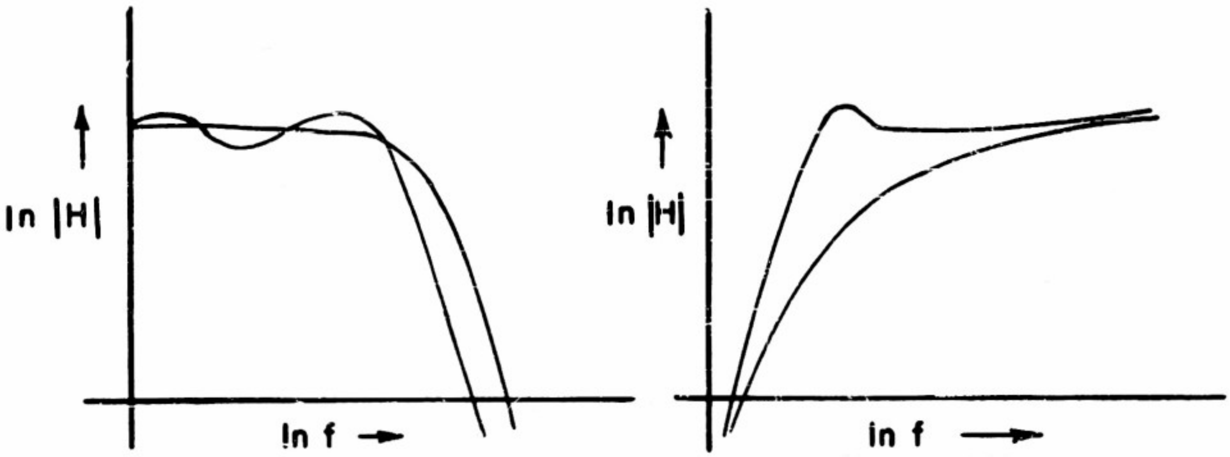
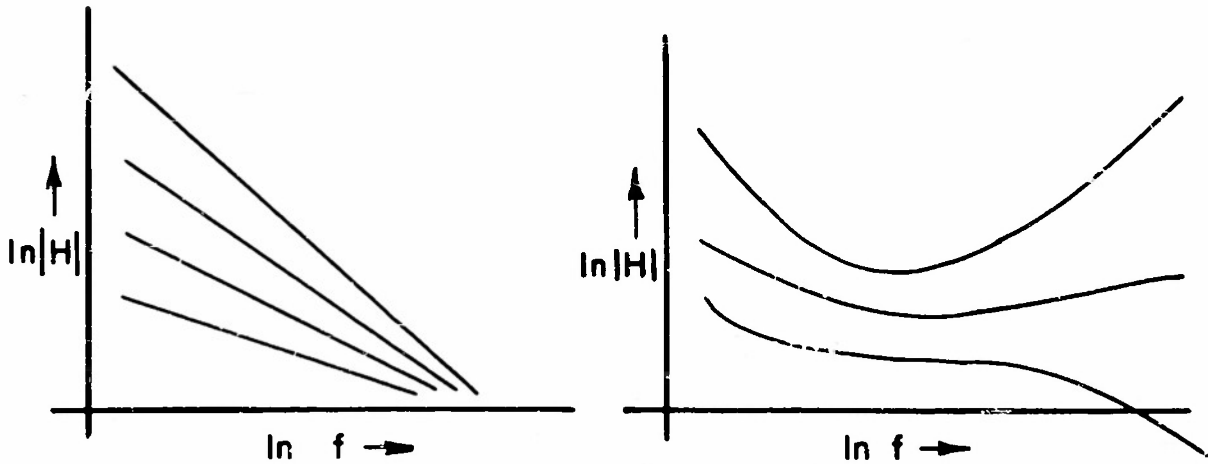
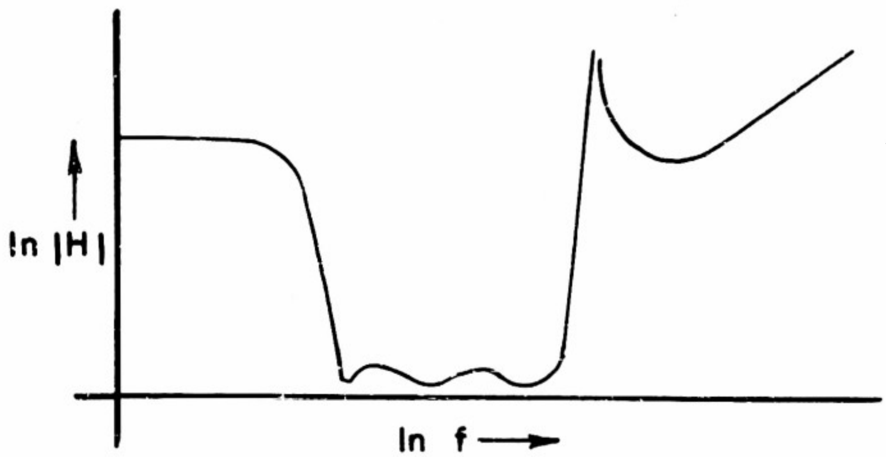


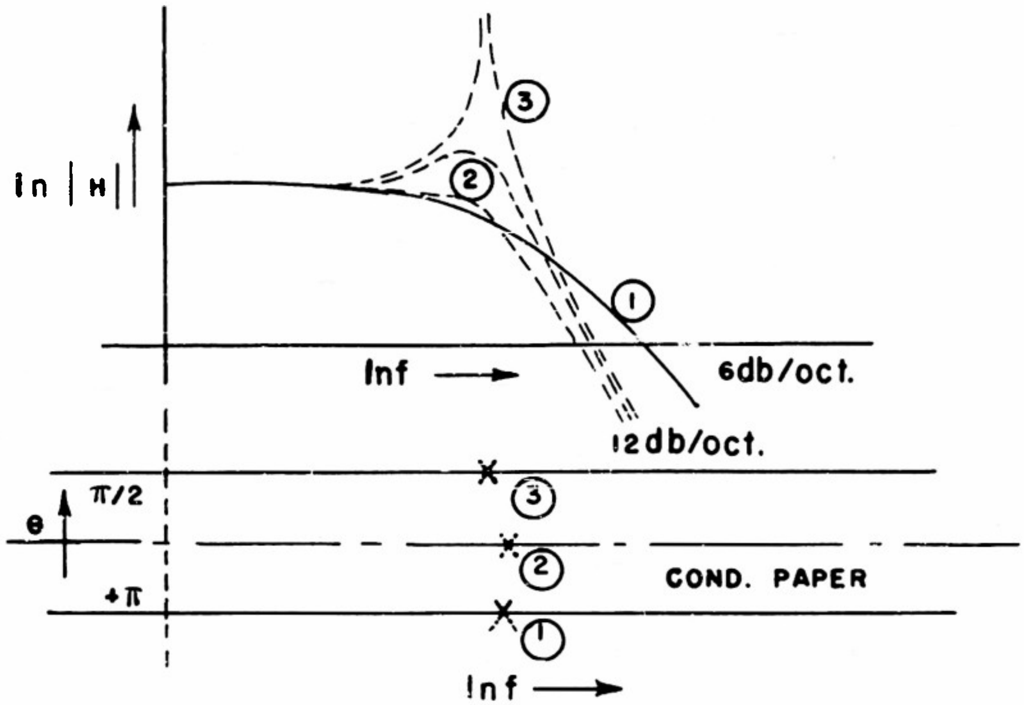
(a)



(b)

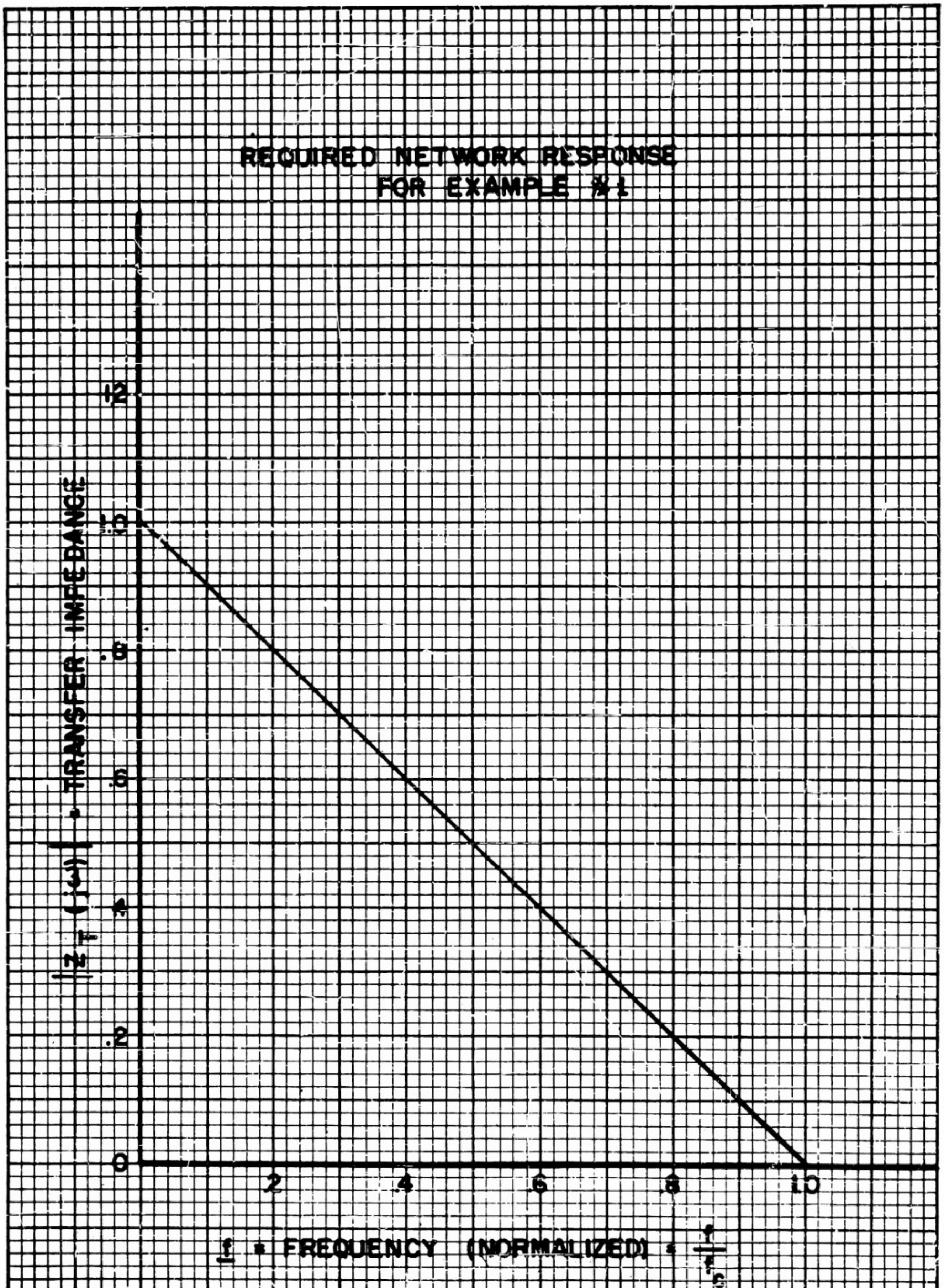


(c)

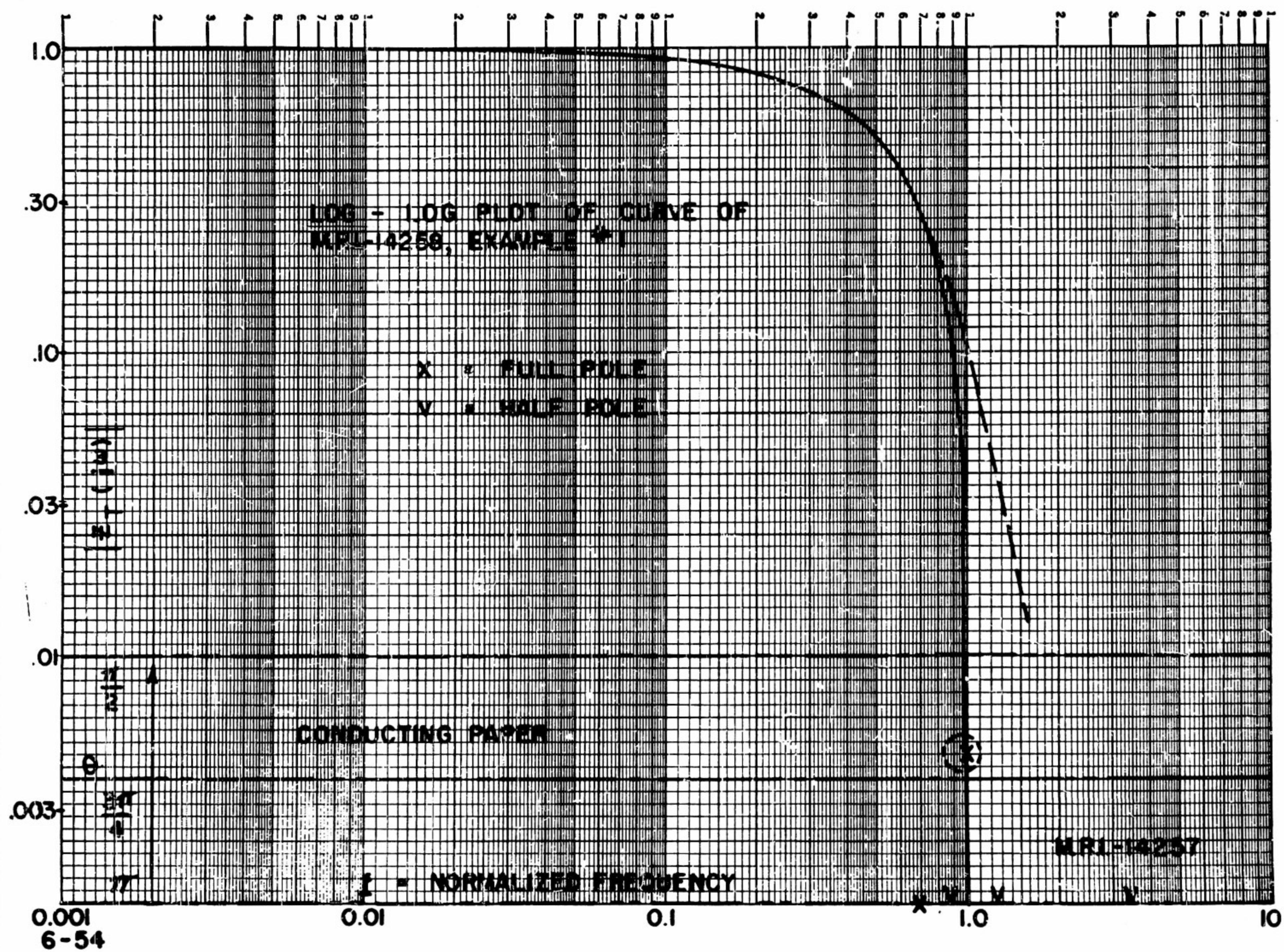


$$H(P) = \frac{1}{\left[P + (\alpha + j\beta)\right]\left[P + (\alpha - j\beta)\right]} \qquad (2)$$

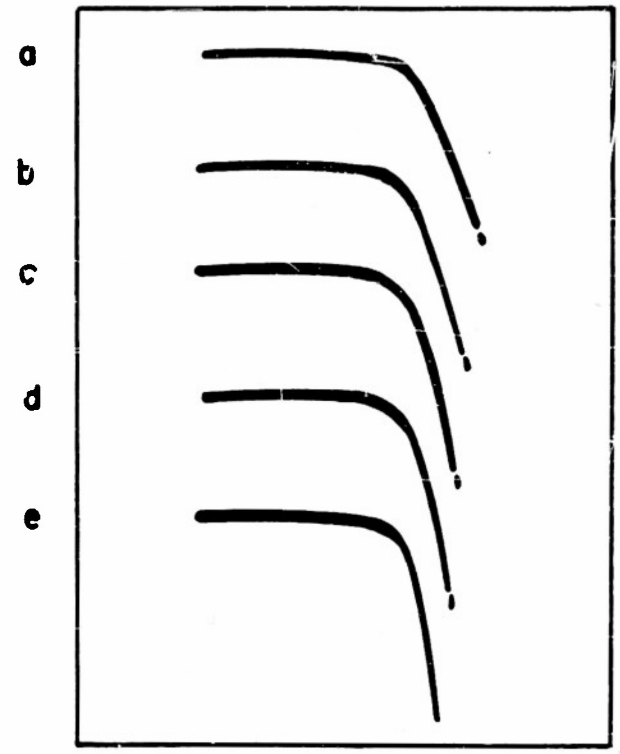
$$\left|H(j\omega)\right|^2 = \frac{1}{\omega^4 + 2(\alpha^2 - \beta^2)\omega^2 + (\alpha^2 + \beta^2)^2} \qquad (3)$$



REQUIRED NETWORK RESPONSE
FOR EXAMPLE #1
$|Z_T(\omega)|$ = TRANSFER IMPEDANCE
1.2
1.0
.8
.6
.4
.2
0
.2
.4
.6
.8
1.0
f = FREQUENCY (NORMALIZED) = $\frac{f}{f_c}$

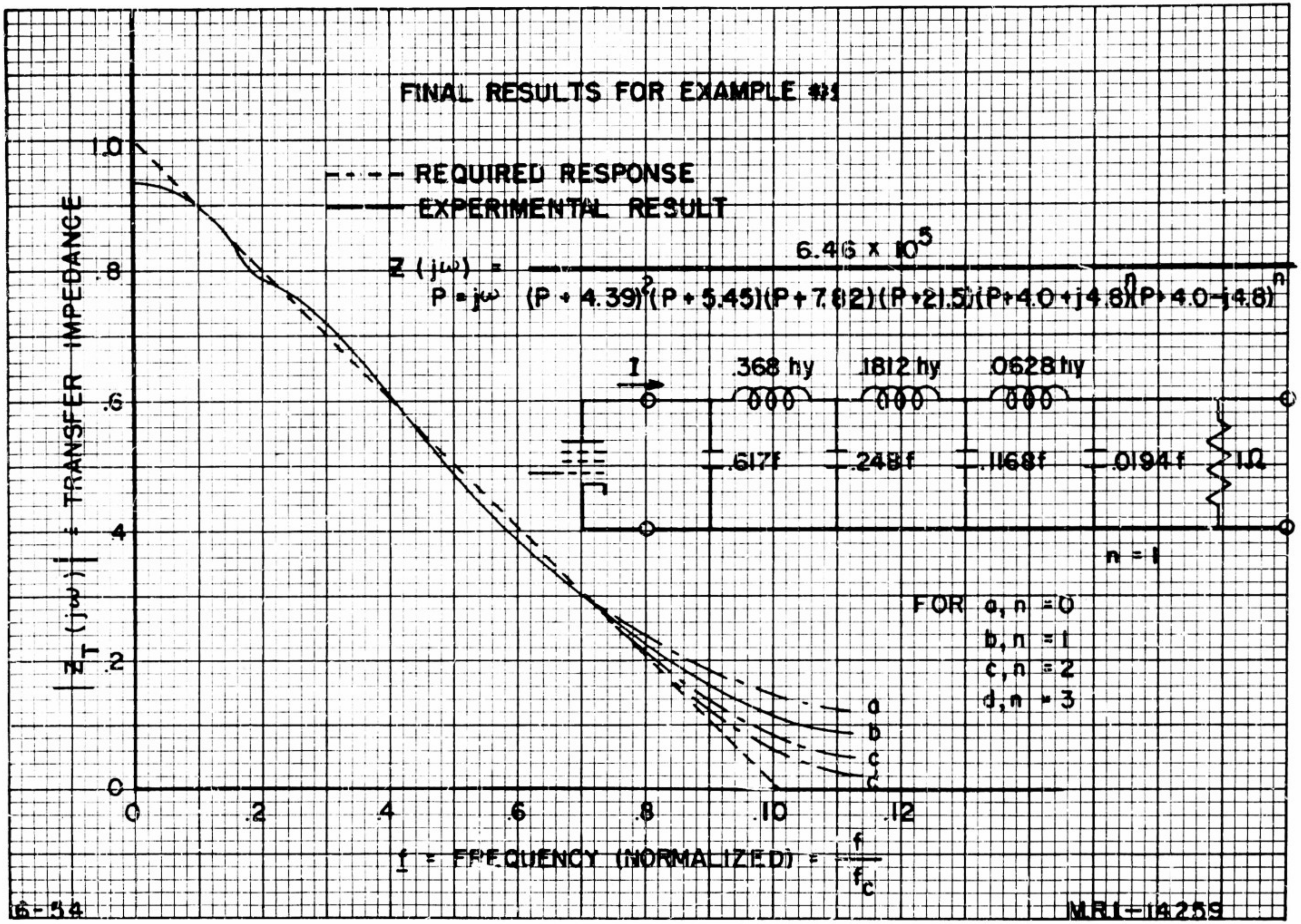


LOG - LOG PLOT OF CURVE OF
MRL-14258, EXAMPLE #1
X = FULL POLE
V = HALF POLE
CONDUCTING PAPER
f = NORMALIZED FREQUENCY
MRL-14257
1.0
.30
.10
.03
.01
.003
0.001
0.01
0.1
1.0
10
6-54

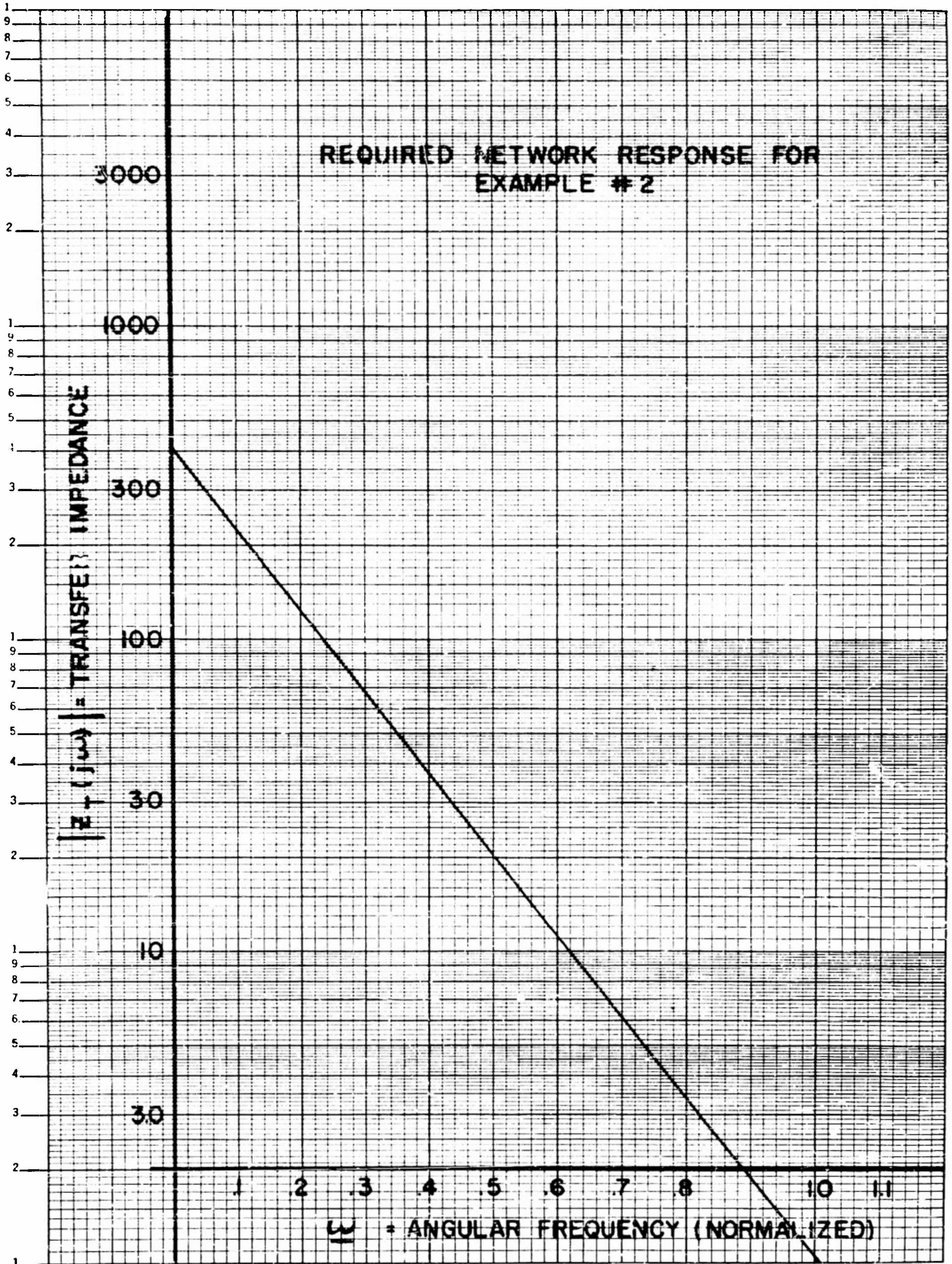


The multiple photographs record the effects of the addition of each singularity in turn from left to right. Curves a, b, c and d are the results of the successive introduction of the poles on the negative real axis (see Fig. MRI-14259). Curve e, the final result, was obtained by adding a pole in position 2, the maximal flatness position. It should be noted that the addition of each singularity does not disturb the lower frequency portions of the response. Increasing the order of the last pole increases the final slope resulting in a better approximation. (See Fig. MRI-14261)

M.R.I.-14258

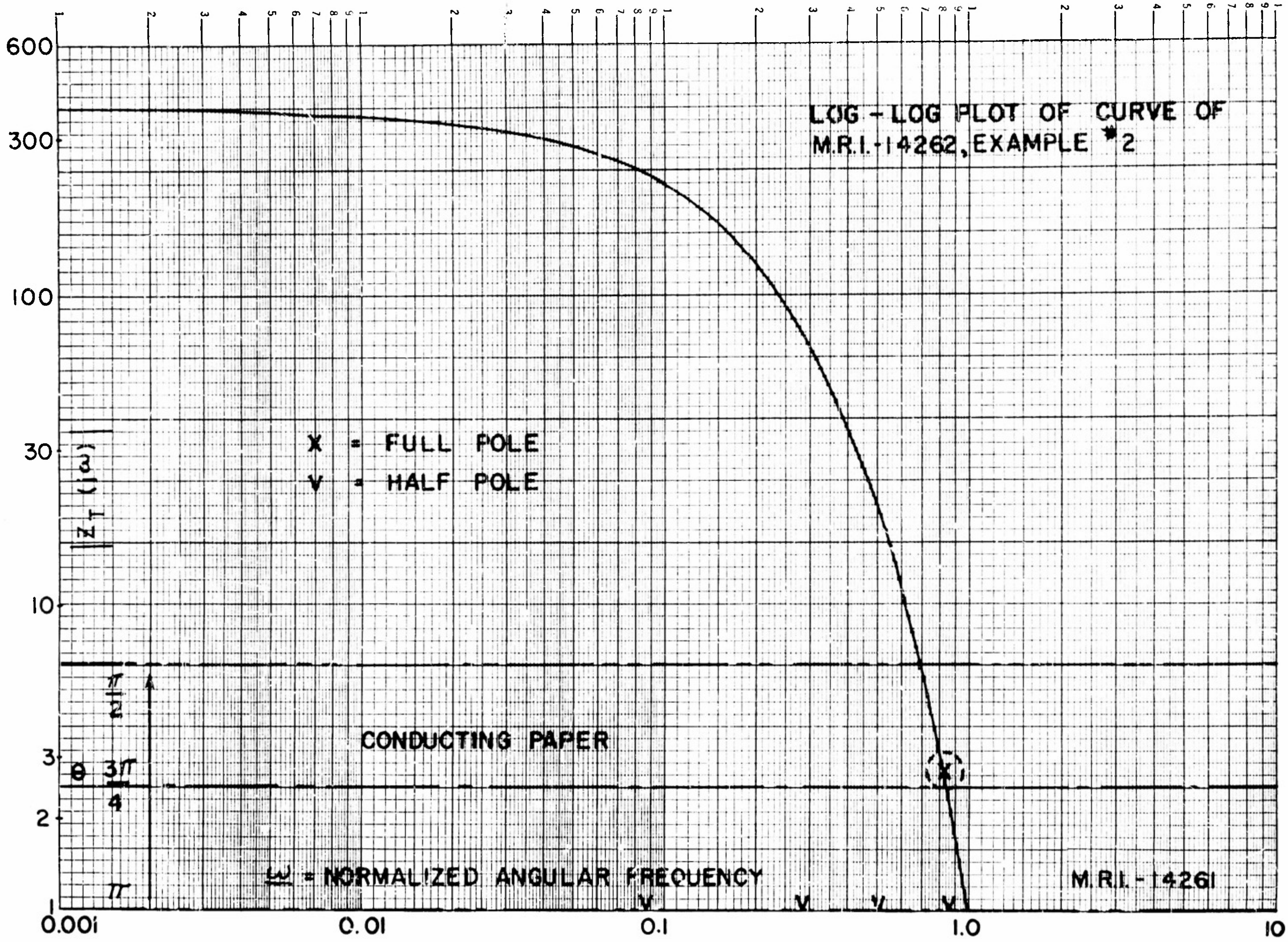


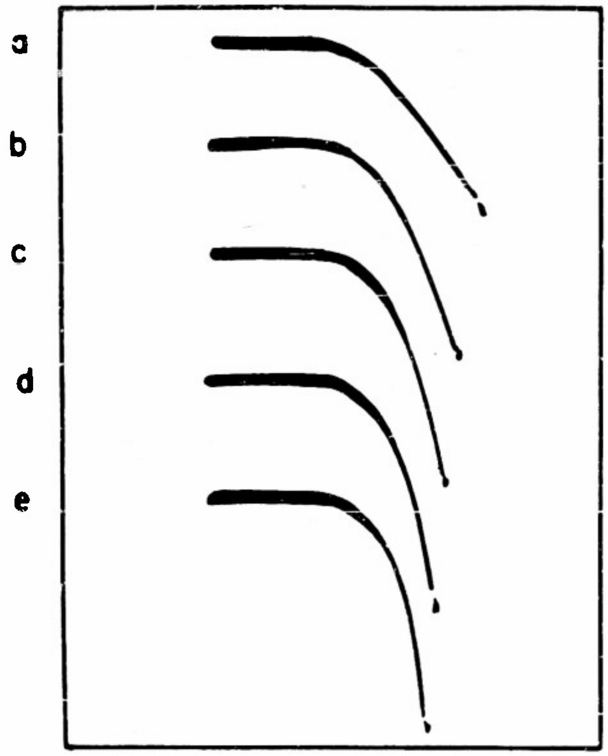
FINAL RESULTS FOR EXAMPLE #1
REQUIRED RESPONSE
EXPERIMENTAL RESULT
Z (jω) = 6.46 x 10^5 / (P + 4.39)^2 (P + 5.45)(P + 7.82)(P + 2.5)^n (P + 4.0 + j4.8)^n (P + 4.0 - j4.8)^n
P = jω
I
.368 hy
.1812 hy
.0628 hy
.617f
.248f
.1168f
.0194f
1Ω
n = 1
FOR a, n = 0
b, n = 1
c, n = 2
d, n = 3
a
b
c
d
|Z_T (jω)| = TRANSFER IMPEDANCE
1.0
.8
.6
.4
.2
0
.2
.4
.6
.8
1.0
1.2
f = FREQUENCY (NORMALIZED) = f / f_c
6-54
MRI-14259



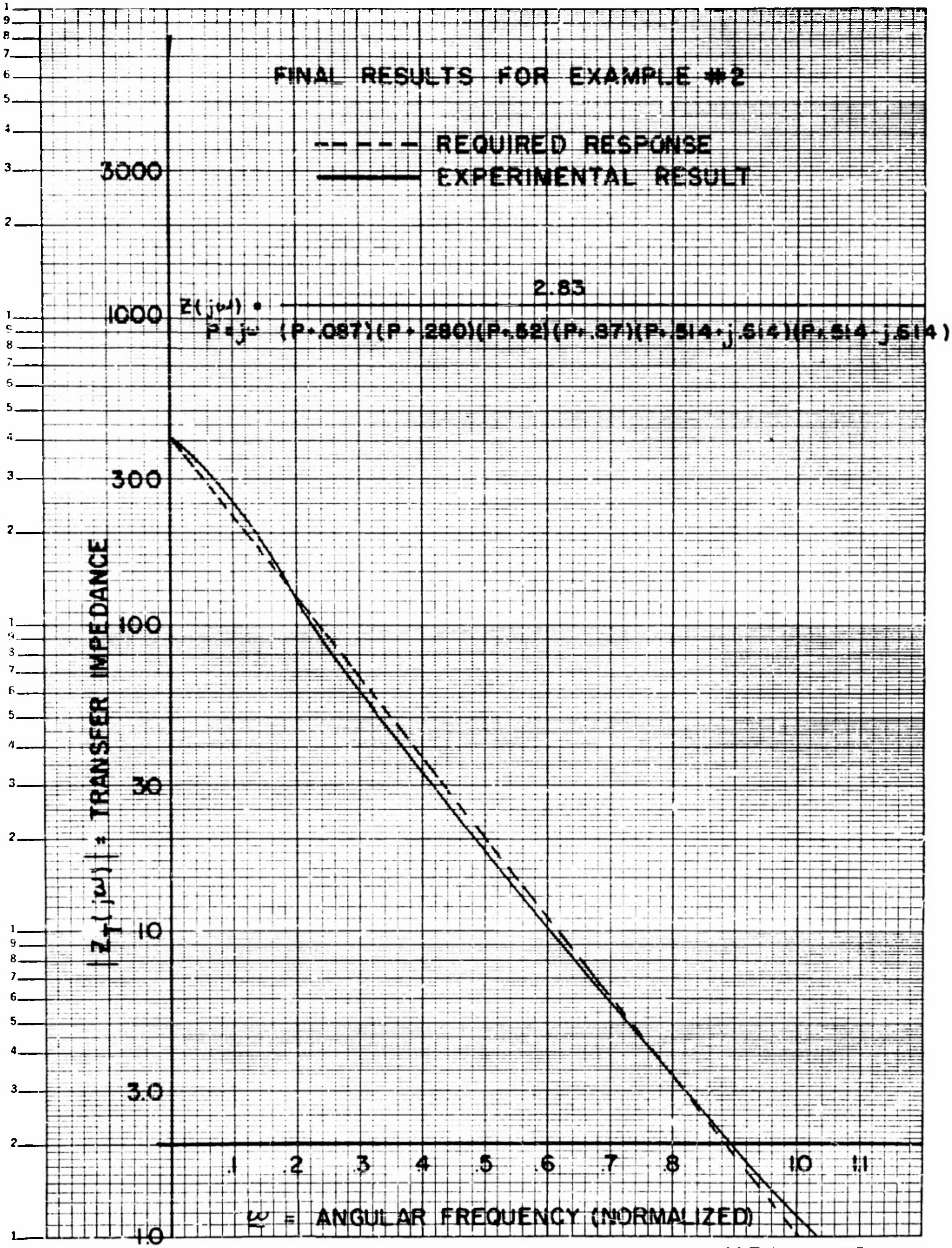
6-54

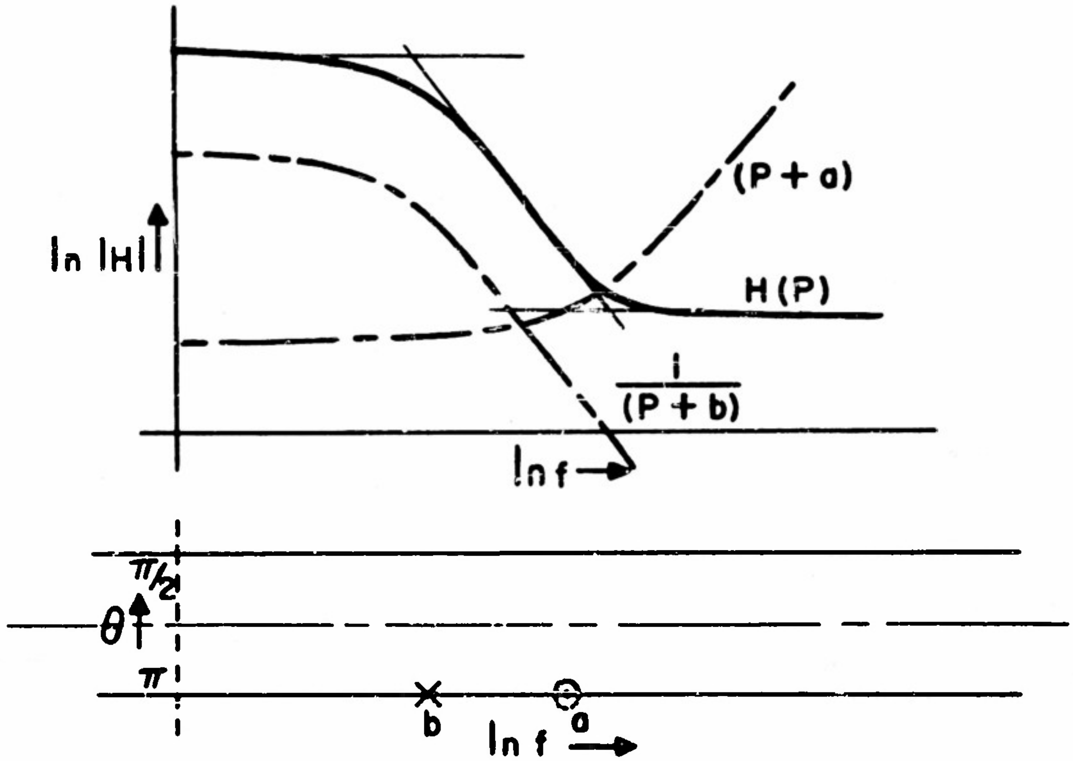
M.R.I.-14260


LOG - LOG PLOT OF CURVE OF
M.R.I.-14262, EXAMPLE #2
X = FULL POLE
V = HALF POLE
CONDUCTING PAPER
ω = NORMALIZED ANGULAR FREQUENCY
M.R.I.-14261
|Z_T (ω)|
θ
π/2
3π/4
π
600
300
100
30
10
3
2
1
0.001
0.01
0.1
1.0
10

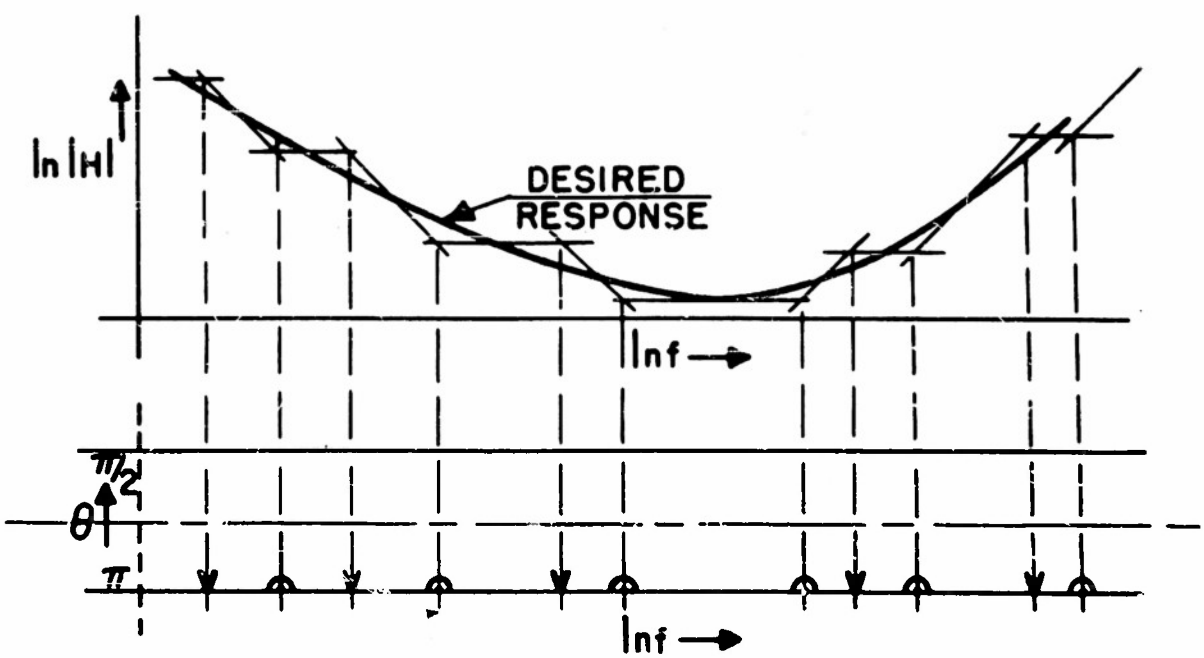


The multiple photographs are, as before, the results of the successive introduction of poles from left to right. Again, curves a-d are due to the singularities on the negative real axis, and curve e, the final response, is obtained from d by adding the singularity encircled in Fig. MRI-14263. The results are similar to those of Fig. MRI-14260.

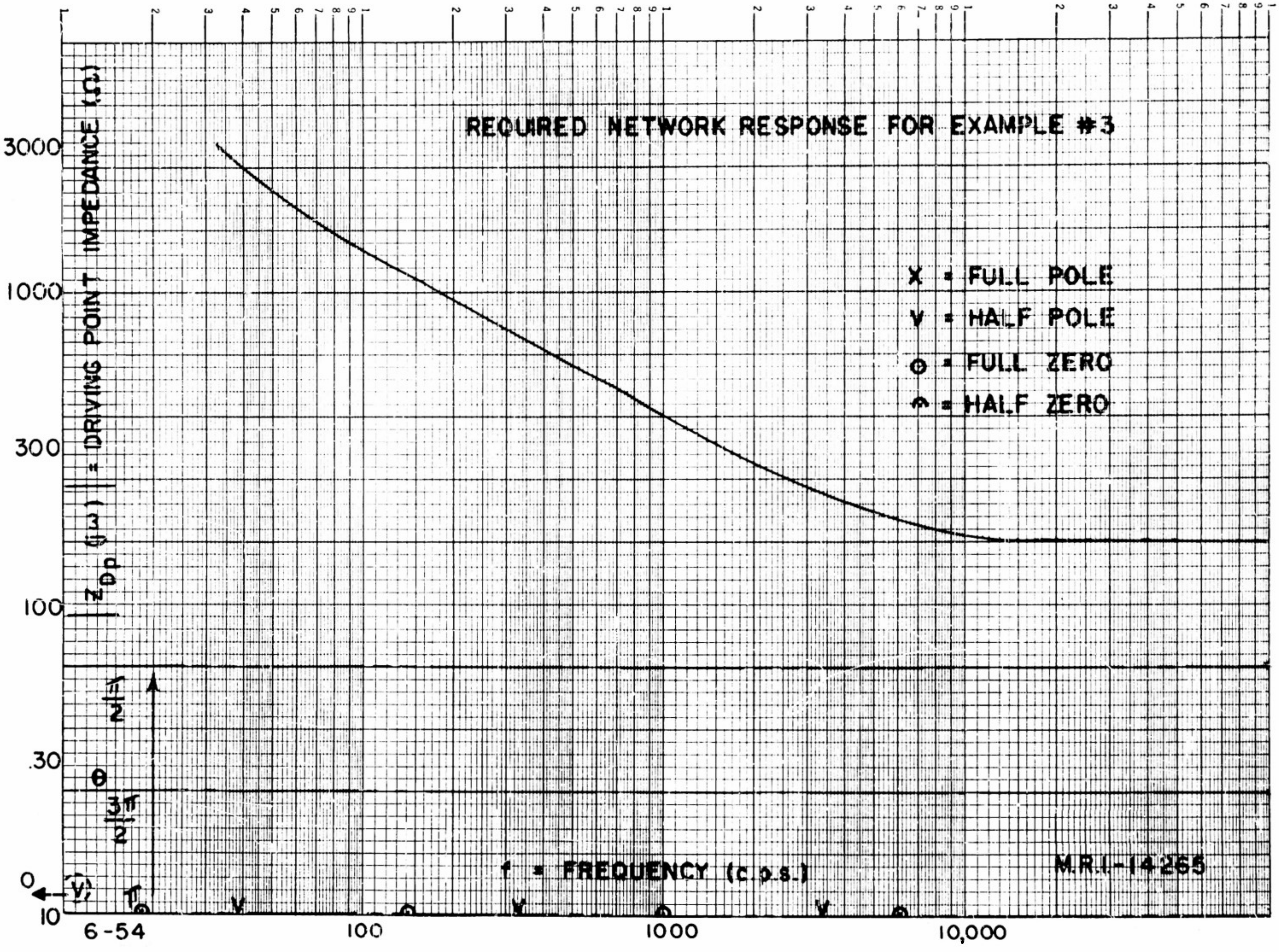

FINAL RESULTS FOR EXAMPLE #2
REQUIRED RESPONSE
EXPERIMENTAL RESULT
Z(jω) = 2.83 / P=jω (P+.087)(P+.280)(P+.52)(P+.87)(P+.514+j.514)(P+.514-j.514)
|ZT(jω)| = TRANSFER IMPEDANCE
3000
1000
300
100
30
10
3.0
1.0
.1
.2
.3
.4
.5
.6
.7
.8
1.0
1.1
ω = ANGULAR FREQUENCY (NORMALIZED)

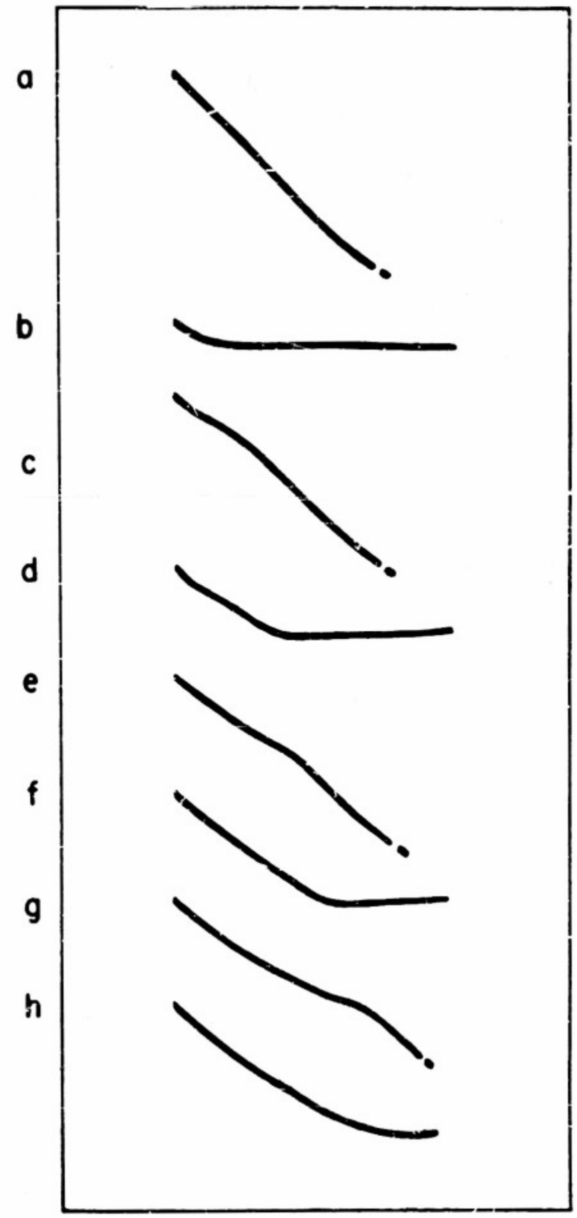


$$H_{(P)} = \frac{(P + a)}{(P + b)}$$

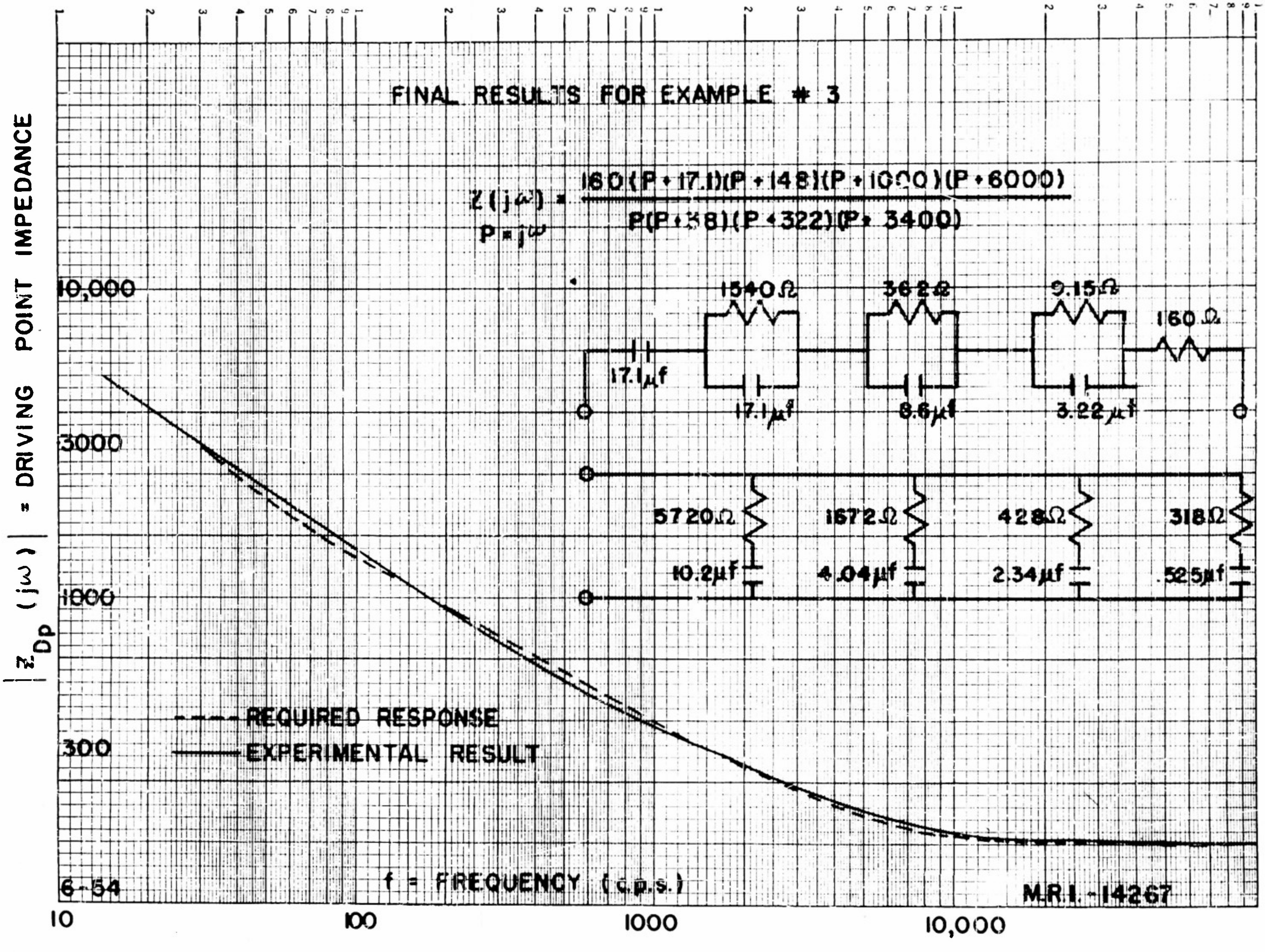


(b)

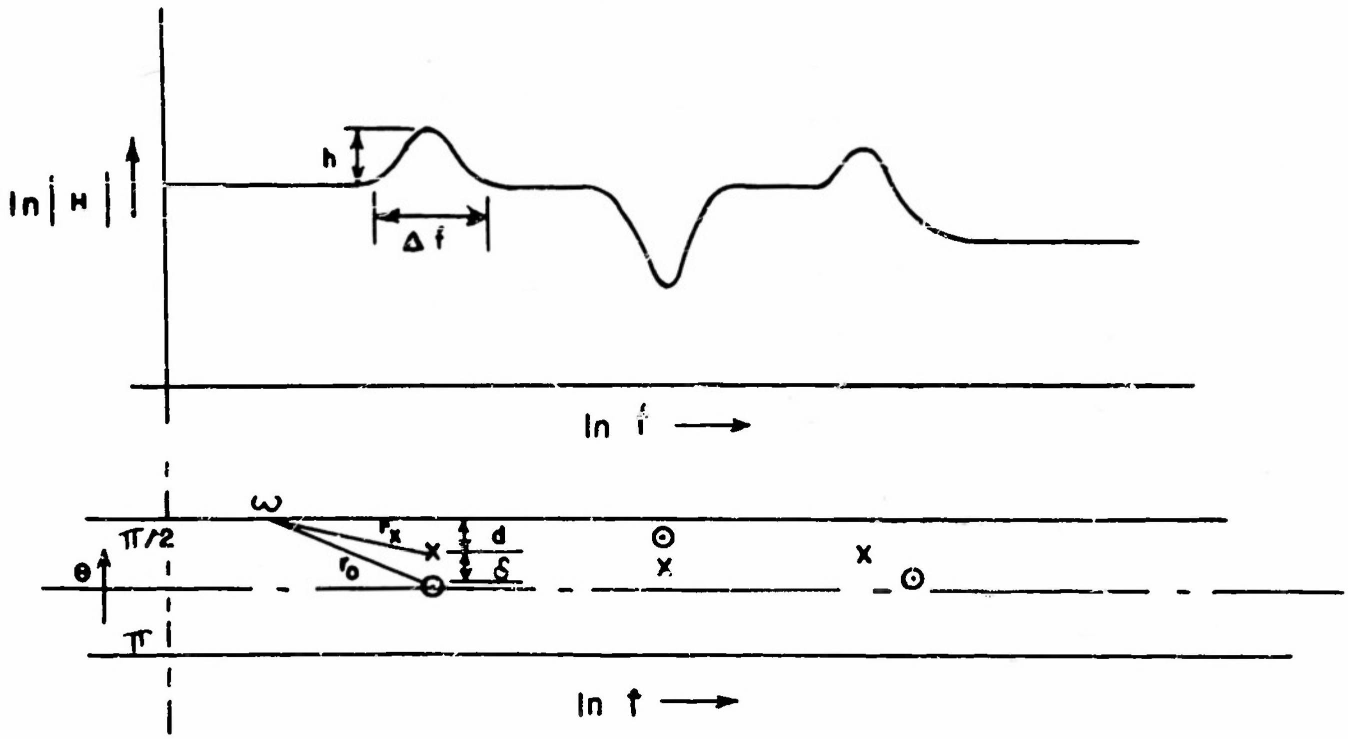

REQUIRED NETWORK RESPONSE FOR EXAMPLE #3
X = FULL POLE
V = HALF POLE
⊖ = FULL ZERO
⌒ = HALF ZERO
|Z_Dp (jω)| = DRIVING POINT IMPEDANCE (Ω)
3000
1000
300
100
30
10
θ
π/2
3π/2
π
0
f = FREQUENCY (c.p.s.)
100
1000
10,000
6-54
M.R.I.-14265

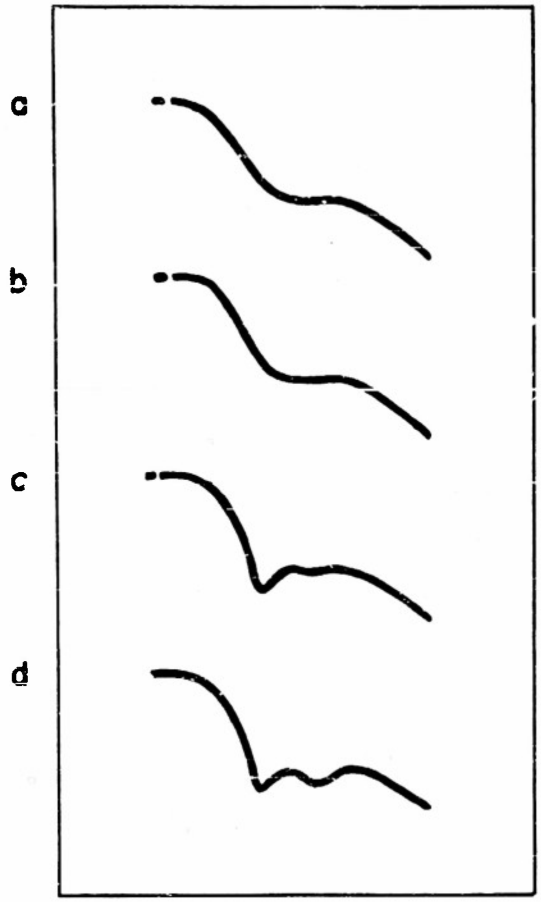


Curve h is the desired response. A pole at the origin results in a 6db/oct. slope (a). The response is then leveled off with a zero (b) and broken again with a pole (c). Continuing in this manner, the curve is forced to oscillate above and below the required response in a step like fashion until the final response is obtained.

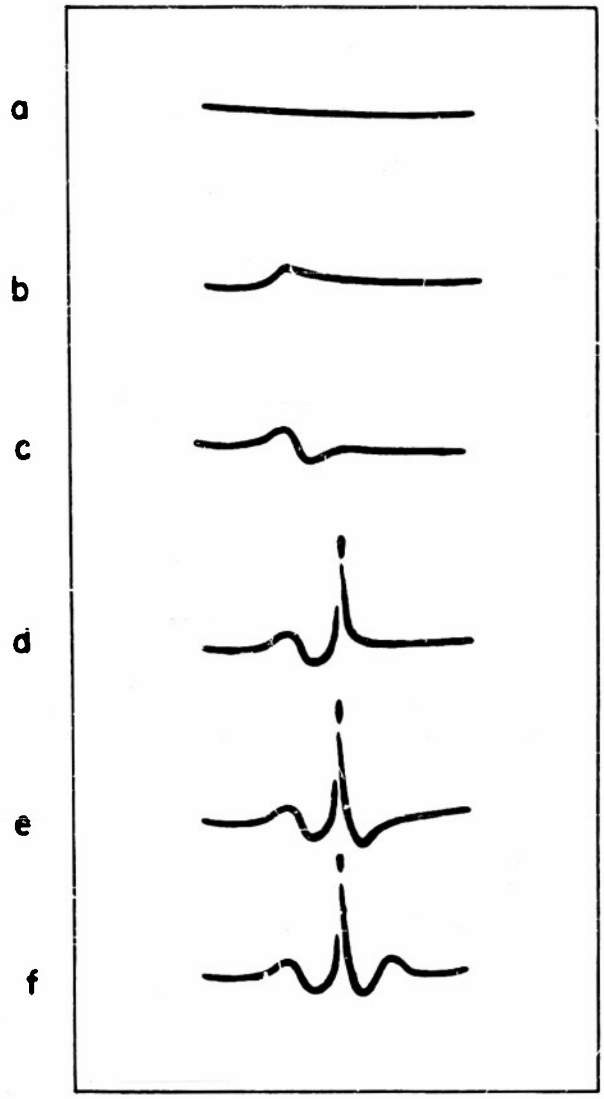


FINAL RESULTS FOR EXAMPLE # 3
Z(jω) = 160(P+17.1)(P+148)(P+1000)(P+6000) / P(P+38)(P+322)(P+3400)
P = jω
1540Ω
362Ω
9.15Ω
160Ω
17.1μf
17.1μf
8.6μf
3.22μf
5720Ω
1672Ω
428Ω
318Ω
10.2μf
4.04μf
2.34μf
.525μf
REQUIRED RESPONSE
EXPERIMENTAL RESULT
10,000
3000
1000
300
|Z_Dp (jω)| = DRIVING POINT IMPEDANCE
f = FREQUENCY (c.p.s.)
10
100
1000
10,000
6-54
M.R.I.-14267

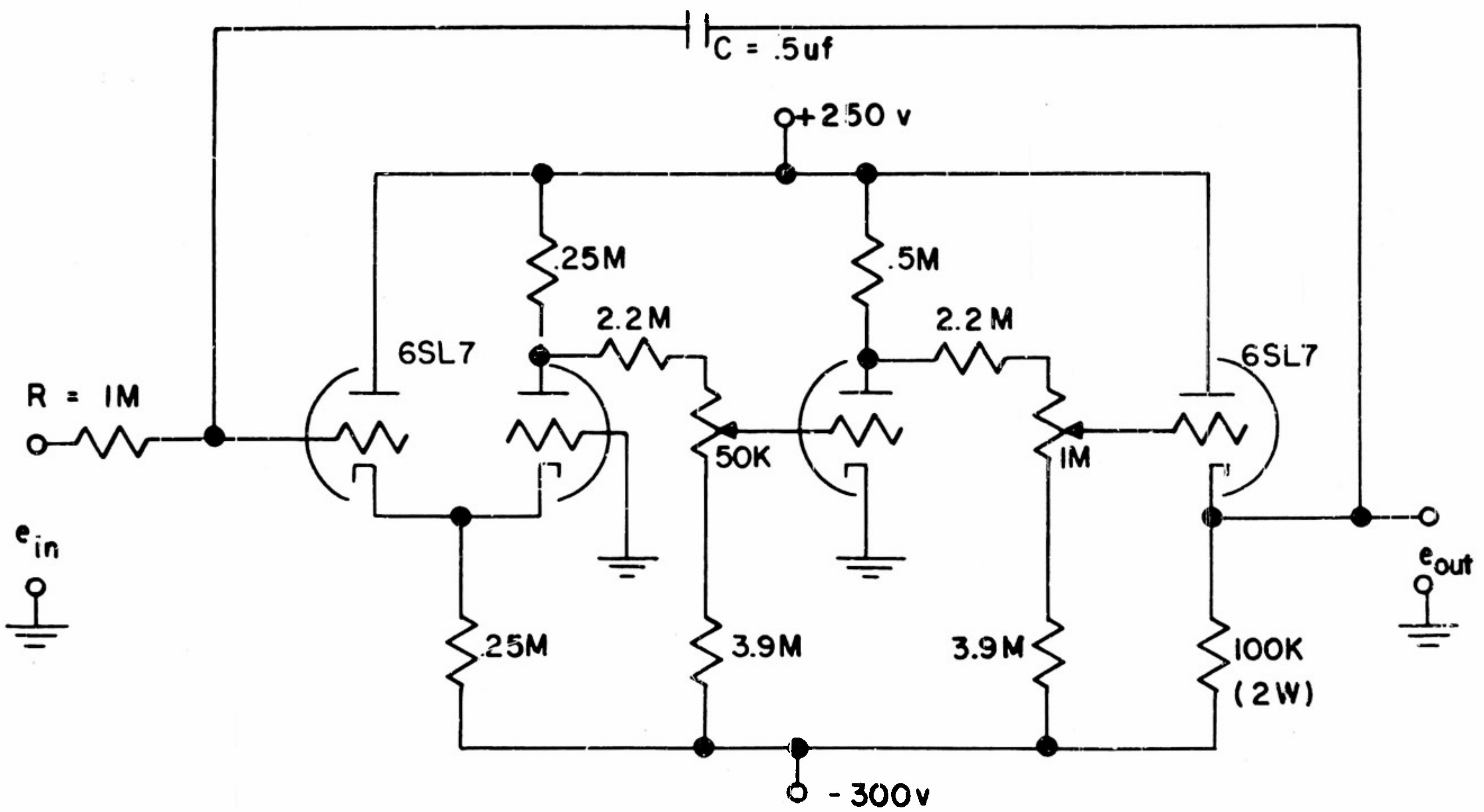

h
ln|H|
Δf
ln f
ω
π/2
θ
π
r_x
r_0
d
δ
ln f



These pictures illustrate the superposition of peaks and valleys on a smooth curve. Curve a is developed with the technique for class (b) curves, that is, with a pole, a zero, and another pole on the negative real axis. All other singularities are introduced as pole-zero pairs. The first pair was introduced, with the pole dominating, in the region of the first break to sharpen the bend (b). The second pair, zero dominating, was responsible for the first valley (c). The following maxima and minima were treated in an identical manner. After the curve was obtained, it was possible to remove several singularities, still maintaining a good approximation by slightly readjusting the remaining poles and zeros.

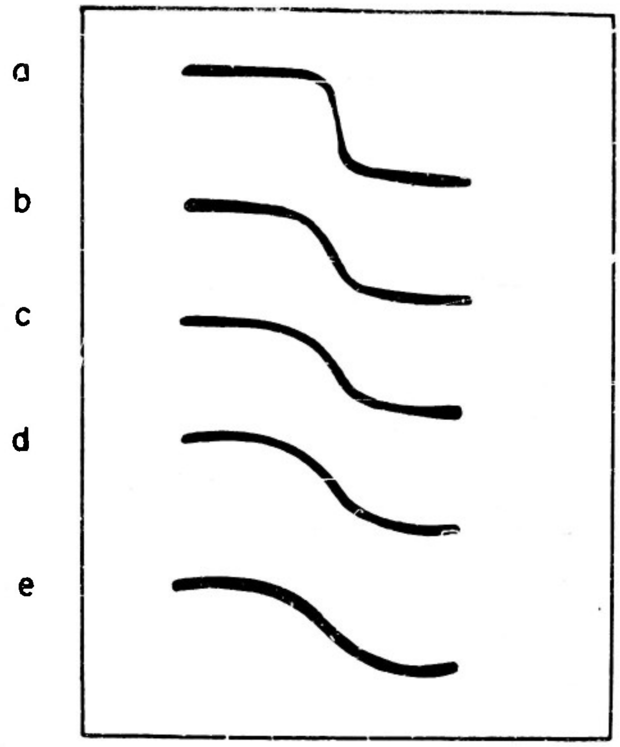


For this type of response, pole-zero pairs are used exclusively to shape the final curve. Proceeding from left to right, and starting with a horizontal line, pole-zero pairs are introduced in line with each peak and valley. The striking conclusion that can be drawn from these photographs is that the addition of singularities in the manner described has no effect on portions of the curve which have already been matched. These pictures illustrate, in an excellent manner, the basic principle upon which the source location techniques were developed.

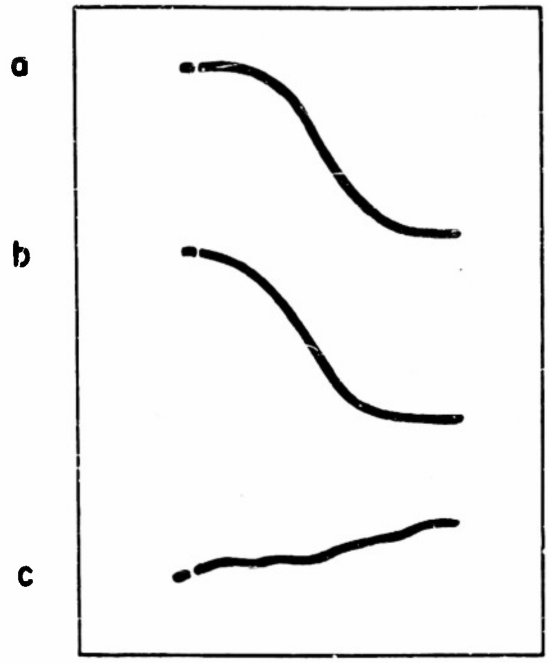


G = AMPLIFIER GAIN $\cong$ 700
RCG = EFFECT. TIME CONST. $\cong$ 6 MIN.
$\frac{1}{RC}$ = OVERALL GAIN = 2

M.R.I.-14271



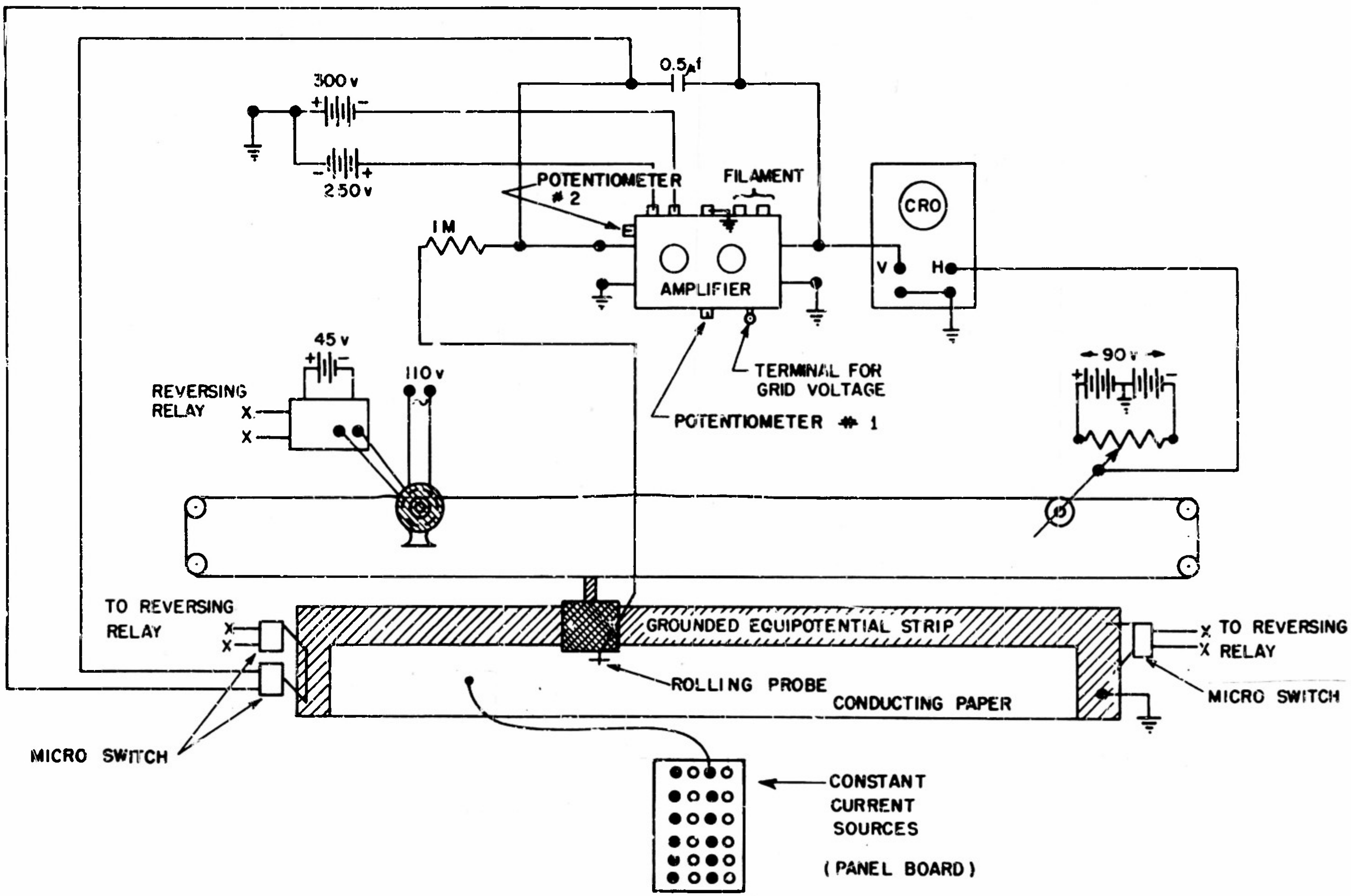
The above curves represent the phase of the input impedance of a parallel RLC circuit for various values of the parameters. Curve a corresponds to a pole almost on the real frequency axis, that is, an almost lossless structure. The other curves, presented in order of increasing loss, indicate the effect of moving the pole along a circular path in the p-plane, since, in order to obtain the remaining response curves, the distance from the axis in the log-plane was varied but the position of the pole relative to the ln(r) coordinate remained fixed. The curves vary from plus to minus ninety degrees. Therefore, any one of the responses can be used for calibration.



Curve a is the phase response for the network synthesized in example No. 1. The curve is asymptotic to + 90° for ω approaching zero and to - 540° for ω going to infinity.

Curve b is the phase response for the transfer impedance of example No. 2. This curve approaches + 90° for ω close to zero and - 450° for ω approaching infinity.

Curve c is the phase of the RC structure synthesized for example No. 3. The response starts at - 90° for ω = 0, and approaches zero for ω going to infinity.



300 v
250 v
0.5µf
1M
POTENTIOMETER # 2
FILAMENT
AMPLIFIER
CRO
V
H
45 v
110 v
REVERSING RELAY
TERMINAL FOR GRID VOLTAGE
POTENTIOMETER # 1
90 v
TO REVERSING RELAY
GROUNDED EQUIPOTENTIAL STRIP
ROLLING PROBE
CONDUCTING PAPER
MICRO SWITCH
CONSTANT CURRENT SOURCES
(PANEL BOARD)

MRI-14274